

Directed Null Transport: Point-to-Point Light Convergence Across Geometries, Dimensions, and Media

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Abstract. Which geometries direct a broad family of light rays from one point to another with synchronized arrival? This paper surveys the mathematics of point-to-point null-geodesic convergence across general relativity, optical geometry, and laboratory analogues, and sharpens the central obstruction: a traversable wormhole throat requires null energy condition violation, while the Raychaudhuri equation shows the same violation actively defocuses null geodesics. We prove the tension as a lemma, show it persists under modified gravity as pure geometry, bound what bounded quantum exotic matter can buy, and exhibit an exact thin-shell construction where refocusing through the throat survives with a quantified focusing budget. Beyond wormholes, we develop anti-de Sitter reconvergence, the solar gravitational lens, rotating and higher-dimensional spacetimes, topological defects, plasma lensing, and laboratory analogues; classify isochronous null refocusing via wiedersehen geometry (resolving the anti-de Sitter case as round-sphere refocusing); and formulate light-transport invertibility as a hierarchy from perfect imaging to information scrambling. Numerical null-geodesic integrations verify synchronized fish-eye refocusing to 10^{-8} and quantify wormhole-throat defocusing. The paper closes by naming the three most promising unexplored directions: the wiedersehen classification under energy conditions, a tabletop synchronized-pulse fish-eye experiment, and the plasma inverse-design problem.

Keywords: general relativity, null geodesics, wormholes, gravitational lensing, optical geometry, Maxwell fish-eye, isochronous refocusing, inverse problems

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1 Introduction

The ellipse has two foci: every ray from one focus reflects to the other, and every such ray has the same optical path length. This paper asks which geometries in general relativity, and which optical media in the laboratory, reproduce that phenomenon for light propagation: a broad family of null geodesics emitted from a point p all arriving at a point q , ideally with equal travel time. We call this *directed null transport* and the equal-time version *isochronous refocusing*.

The question matters for three reasons. First, it is mathematically sharp: it reduces, via Fermat’s principle, to a question about geodesic refocusing in Riemannian optical geometry, where the classification theory of *wiedersehen* manifolds gives real traction. Second, it exposes a precise obstruction at the intersection of traversability and focusing: the exotic matter that holds a wormhole throat open is, by the Raychaudhuri equation, the agent that defocuses light there. Making that tension quantitative is the paper’s analytical core. Third, the design goal has known partial realizations, from anti-de Sitter reconvergence to the solar gravitational lens to tabletop fish-eye devices, and surveying them against common criteria reveals which directions are genuinely unexplored.

What this paper contributes:

1. A proved tension lemma: at any Morris-Thorne throat satisfying flare-out, the null convergence condition fails on radial congruences in a neighborhood of the throat, and the throat converts convergence into divergence (Section 2).
2. A no-go result for modified gravity: the defocusing sign $R_{ab}k^ak^b < 0$ at a flared throat is pure geometry (the trace term drops out for null vectors), so $f(R)$ and Einstein-Gauss-Bonnet theories relabel but do not remove the obstruction (Section 6).
3. An exact thin-shell construction where refocusing through the throat survives: two S^3 caps with exactly fish-eye optical geometry joined at a shell, bulk matter satisfying the null energy condition, and a proved focusing budget governing when the shell’s diverging kick is overcome (Section 6).
4. Quantitative bounds on bounded exotic matter: Casimir energy falls short of a meter-scale throat by 10^{46} , and an exact relation between exotic cost and defocusing kick (Section 6).
5. A resolution of the anti-de Sitter case: its optical metric is the open hemisphere of the round sphere, unifying AdS, the Einstein static universe, and the fish-eye as round-sphere *wiedersehen* refocusing; plus a new non-round NEC-satisfying family with exact isochronous refocusing (Section 12).
6. New results on rotating and defect spacetimes: synchronous half-cone refocusing at the causal boundary of Gödel-type universes; an infinite winding image tower around cosmic strings with proved generic asynchrony (Sections 7, 9).
7. An invertibility hierarchy ranking all studied geometries by the data needed to invert light transport, from perfect imaging to information scrambling (Section 13).
8. Numerical null-geodesic integrations verifying fish-eye synchronization to 10^{-8} and quantifying throat defocusing (Section 14).
9. A concrete tabletop experimental proposal with falsifiable predictions (Section 11).

Status conventions are used throughout: THEOREM (proved, cited), COMPUTATION (derived here, method stated), LEMMA (proved here), CONJECTURE and OPEN PROBLEM (unresolved). Quantitative claims carry the evidence vocabulary Observed / Reconstructed / Modelled / Assumed with grades A/B/C. Signature $(-, +, +, +)$ and $G = c = 1$ throughout.

The rest of the paper proceeds as follows. Section 2 develops the mathematical preliminaries and proves the tension lemma. Section 3 treats perfect imaging in optical geometry. Section 4 covers cosmological refocusing (AdS, Einstein static universe) and Section 5 astrophysical lensing (the solar gravitational lens). Section 6 is the wormhole chapter: the obstruction and four candidate resolutions. Sections 7–10 survey rotating spacetimes, higher dimensions, topological defects, and plasma media. Section 11 covers laboratory analogues and the experimental proposal. Section 12 classifies isochronous null refocusing. Section 13 develops the inverse problem and invertibility hierarchy. Section 14 presents the numerical experiments. Section 15 states falsifiers and limitations, and Section 16 names the three most promising unexplored directions.

2 Mathematical preliminaries: the design goal and its central tension

2.1 Perfect and isochronous refocusing

Definition 2.1 (Perfect and isochronous refocusing). Let p, q be distinct points of a spacetime. *Perfect refocusing* at (p, q) : every null geodesic through p passes through q . *Isochronous refocusing* at (p, q) : perfect refocusing holds, and all such null geodesics reach q with equal Killing time (for static observers).

The ellipse analogy is exact in one respect and misleading in another. Exact: the equal-path-length property of the ellipse is isochrony. Misleading: reflection is not available in Lorentzian geometry; convergence must come from curvature or refractive structure.

2.2 Fermat's principle and the optical metric

Theorem 2.2 (Fermat reduction, static case (Perlick, 2000)). Let (M, g) be static, $ds^2 = -e^{2\Phi(x)}dt^2 + h_{ij}(x)dx^i dx^j$. Null geodesics between fixed spatial endpoints stationarize the Killing arrival time $T = \int e^{-\Phi} \sqrt{h_{ij} dx^i dx^j}$. Their spatial projections are geodesics of the optical (Fermat) metric $\tilde{h}_{ij} = e^{-2\Phi} h_{ij}$, and Killing time coincides with optical arc length along each ray.

Isochronous refocusing in spacetime is therefore equal-length geodesic refocusing in $(\tilde{\Sigma}, \tilde{h})$. COMPUTATION (grade A): for the Morris-Thorne metric $ds^2 = -e^{2\Phi(r)}dt^2 + dr^2/(1 - b(r)/r) + r^2 d\Omega^2$,

$$\tilde{h} = e^{-2\Phi(r)} \left[\frac{dr^2}{1 - b(r)/r} + r^2 d\Omega^2 \right]. \quad (1)$$

2.3 Flare-out forces NEC violation

In the orthonormal frame, the Einstein components give $\rho = b'/8\pi r^2$ and

$$\rho + p_r = \frac{1}{8\pi r^2} \left[b' - \frac{b}{r} + 2(r - b)\Phi' \right]. \quad (2)$$

At the throat $r = b_0$ where $b(b_0) = b_0$, the flare-out condition $b'(b_0) < 1$ yields

$$\rho + p_r|_{r=b_0} = \frac{b'(b_0) - 1}{8\pi b_0^2} < 0, \quad (3)$$

so for every radial null vector, $T_{ab}k^ak^b = (\rho + p_r)(k^t)^2 < 0$: the NEC is violated at the throat (Morris and Thorne, 1988). Since k^a is null, $R_{ab}k^ak^b = 8\pi T_{ab}k^ak^b < 0$ there as well.

2.4 Null Raychaudhuri and the focusing theorem

For a null congruence with expansion θ , shear σ_{ab} , twist ω_{ab} ,

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{ab}k^ak^b. \quad (R)$$

Theorem 2.3 (Null focusing (Hawking and Ellis, 1973; Wald, 1984)). *If $R_{ab}k^ak^b \geq 0$ for all null k^a and $\omega_{ab} = 0$, then $\theta(0) < 0$ implies $\theta \rightarrow -\infty$ within affine parameter $2/|\theta_0|$: a conjugate point is forced.*

Where $R_{ab}k^ak^b < 0$, writing $Q = -R_{ab}k^ak^b > 0$, the curvature term becomes a positive source for θ : NEC violation is an anti-focusing agent, and where $Q > \theta^2/2 + \sigma^2 - \omega^2$ convergence is actively reversed.

2.5 The tension lemma

Lemma 2.4 (Throat anti-focusing; proved). *Let (M, g) be Morris-Thorne with Φ' bounded near the throat $r = b_0$ and flare-out $(b - b'r)/2b^2 > 0$ at $r = b_0$. Then: (a) $\rho + p_r < 0$ and $R_{ab}k^ak^b < 0$ for radial null k^a in a neighborhood $b_0 \leq r < b_0 + \epsilon$; (b) the focusing theorem is inapplicable on every radial null congruence there; (c) for the affinely parametrized radial congruence, $\theta = \pm(2E/r)e^{-\Phi}\sqrt{1 - b/r}$, so $\theta|_{r=b_0} = 0$ with $d\theta/d\lambda|_{r=b_0} = Q > 0$: the throat converts convergence into divergence.*

Proof. Part (a): $\rho + p_r = [b' - b/r + 2(r - b)\Phi']/8\pi r^2$ is strictly negative at $r = b_0$ by flare-out, and $2(r - b)\Phi'$ vanishes continuously there, so negativity persists nearby; $R_{ab}k^ak^b = 8\pi(\rho + p_r)(k^t)^2$ for radial null k^a . Part (b) is the contrapositive of the focusing theorem. Part (c): with $k^a = E(e^{-2\Phi}, \pm e^{-\Phi}\sqrt{1 - b/r}, 0, 0)$ and $\sqrt{-g} = e^{\Phi}r^2 \sin\theta(1 - b/r)^{-1/2}$, $\theta = \nabla_a k^a = \pm(2E/r)e^{-\Phi}\sqrt{1 - b/r}$; spherical symmetry gives $\sigma_{ab} = \omega_{ab} = 0$, so (R) at the throat is $d\theta/d\lambda = -R_{ab}k^ak^b = Q > 0$. $\square$

In the language of Hochberg and Visser (1998), the throat is a marginally anti-trapped surface. The exotic matter does not merely permit the throat; in the Raychaudhuri equation it is the agent that un-focuses light there. This is the paper's central obstruction, and Section 6 attacks it directly.

3 Perfect imaging in optical geometry

The Maxwell fish-eye lens has refractive index

$$n(r) = \frac{2n_0}{1 + (r/a)^2}, \quad (4)$$

and is an absolute optical instrument: every ray from a point refocuses at the antipodal point, and all rays have equal optical path $\pi n_0 a$ (Born and Wolf, 1999; Leonhardt, 2009). The geometry is the stereographic projection of the 3-sphere: rays are great circles, and the equal-path property is the statement that all great-circle arcs between antipodes have length π . The Luneburg lens is the second classical absolute instrument (Luneburg, 1964).

The general-relativistic reading uses Gordon’s optical metric (Gordon, 1923): a static spacetime acts as a refractive medium with effective index $n = e^{-\Phi}$ (in the ultrastatic case, the spatial metric itself). COMPUTATION (grade A): the fish-eye index profile is realized exactly as the Einstein static universe in isotropic coordinates (Visser, 2013); i.e., one absolute optical instrument is physically realized with normal matter (dust plus $\Lambda > 0$), satisfying the NEC. The Schwarzschild refractive index is $n = (1 + M/2r)^3/(1 - M/2r)$ (de Felice, 1971).

Leonhardt’s perfect-imaging program (Leonhardt and Philbin, 2006; Leonhardt, 2009) and transformation optics translate directly: “perfect imaging by a spacetime” means its optical metric is isometric, up to the conformal factor, to an absolute-instrument geometry. The known limitation carries over: perfect imaging in the monochromatic sense does not imply equal travel time; isochrony is the stronger condition classified in Section 12.

4 Cosmological refocusing: anti-de Sitter and the Einstein static universe

Theorem 4.1 (AdS reconvergence). *In global anti-de Sitter space, every null geodesic emitted from a boundary point reconverges at the antipodal boundary point after Killing time $\Delta\tau = \pi$ (in AdS units), independent of initial direction.*

The proof is via conformal compactification: AdS is conformal to a region of the Einstein static universe, and null geodesics map to great-circle arcs (Maldacena, 1998; Hubeny, Liu, and Rangamani, 2007). This is the cleanest known example of synchronized null refocusing, and it plays a structural role in AdS/CFT through bulk-point singularities.

In the Einstein static universe, null geodesics refocus at the antipode after $\Delta t = \pi R_E$, where R_E is the curvature radius (Einstein, 1917). The price is well known: the model is unstable (Eddington, 1930) and non-expanding, so this is mathematical existence, not astrophysics. Both examples satisfy the NEC.

The optical-geometry design program asks which spatial optical metrics arise as constant-time slices of physical spacetimes and what stress-energy they require. Given a desired index profile $n(x)$, the Einstein equations formally determine T_{ab} , but no general theorem characterizes which profiles admit NEC-satisfying realizations; the fish-eye/ESU identification above is the principal positive example, and the classification of Section 12 gives the sharpest known positive results.

5 Astrophysical lensing: the solar gravitational lens

The Sun focuses light from distant sources, but onto a focal *line*, not a point. COMPUTATION (Modelled, B): for impact parameter b , the focal distance is $z(b) = b^2/2r_g$ with r_g the solar gravitational radius; the minimum, at the solar limb $b = R_\odot$, is $z_{\min} = R_\odot^2/2r_g \approx 548$ AU (Eshleman,

1979; Turyshchev, 2017). Only the one-parameter family of rays grazing the limb participates: this is annular focusing, not broad-family refocusing. By axisymmetry, each ring is internally synchronized, but different rings focus at different distances: there is no single exit point and no global isochrony.

The fidelity limits are quantified: coronal plasma makes the lens strongly chromatic at radio wavelengths (negligible at optical) (Turyshchev and Toth, 2019); the solar quadrupole moment J_2 turns the point focus into an astroid caustic (Einstein cross pattern) (Turyshchev and Toth, 2021); solar motion and the $b \geq R_\odot$ occultation cut complete the error budget. The predicted gain is $\sim 10^{11}$ at $1 \mu\text{m}$ (Turyshchev, 2017), and NASA NIAC Phase III funding (2M, 2020) supported mission studies. Verdict: astrophysically real, no exotic matter, but a focal line rather than a point and only a one-parameter ray family. It is the closest realized system to the design goal and the measure of how far “real” falls short of “perfect.”

6 Wormholes: the obstruction and four candidate resolutions

The tension lemma makes the wormhole case the sharpest test of the design goal: can light be refocused *through* a region whose curvature actively defocuses it? Four avenues are developed; each gets an honest verdict.

6.1 Avenue (a): thin-shell spatial separation

Confine the exotic matter to an infinitesimal shell at the throat (Visser’s thin-shell construction (Visser, 1989a,b, 1995)) and do the refocusing in the exteriors, which contain only normal matter. The Israel-Darmois junction conditions give the Lanczos equation $S_j^i = -(1/8\pi)([K_j^i] - \delta_j^i[K])$ for the surface stress-energy (Israel, 1966; Lanczos, 1924; Darmois, 1927).

Theorem 6.1 (Exotic shell is generic). *For every static spherically symmetric thin-shell wormhole, the surface energy density satisfies $\sigma_s < 0$: each exterior contributes positively to $[K^\theta_\theta]$, the thin-shell form of flare-out.*

COMPUTATION (grade A): for Minkowski exteriors, $\sigma_s = -1/2\pi a$, $p_s = 1/4\pi a$ at throat radius a .

The junction conditions price but do not forbid fish-eye exteriors: $[h_{ij}] = 0$ requires only a common areal radius, and Lanczos then dictates the required (σ_s, p_s) . COMPUTATION (grade A): an exact “Einstein-static fish-eye wormhole”: two S^3 caps joined at a shell, whose optical metric is *exactly* the Maxwell fish-eye (round S^3), with bulk matter $\rho = 3/8\pi R^2$, $p = -\rho/3$ satisfying NEC/WEC/DEC, so the *only* exotic matter is the shell, with $\sigma_s = -\cot \chi_0/2\pi R$.

The critical question is what the shell does to crossing rays:

Lemma 6.2 (Shell crossing; proved). *Individual null geodesics cross a timelike thin shell with continuous tangent: no Snell’s-law kink. (Contrast null shells, which do refract (Barrabès and Israel, 1991).)*

Theorem 6.3 (Expansion jump; proved). *Integrating Raychaudhuri across the distributional curvature gives $\Delta\theta = -8\pi S_{ab}k^ak^b/|k \cdot n|$. For radial crossing $\Delta\theta > 0$ always (since $\sigma_s < 0$); in the symmetric case $\theta^+ = -\theta^-$: the shell mirror-reflects the expansion. Non-radial rays receive an angle-dependent kick: shell-induced aberration.*

The shell is a finite diverging thin-lens element, not a refractive interface; caustic loci are fixed by the C^0 metric, so the shell cannot move or erase a focus.

Theorem 6.4 (Focusing budget; proved). *A caustic forms through the shell iff the post-shell integrated Ricci focusing exceeds the finite kick. This is the precise “it depends on X .”*

COMPUTATION (grade A): exact pole-to-pole refocusing through the throat in closed form, $\theta(\chi) = -(2/R) \cot \chi$, with the kick $\Delta\theta = 4 \cot \chi_0/R$ explicitly overcome and $\theta \rightarrow -\infty$ at the image point. Verdict: avenue (a) *can work*. The finite kick, versus a continuous NEC-violating drain, is the structural payoff of spatial separation. Open: off-axis aberration compensation, asymptotically flat fish-eye-like exteriors, and shell stability (the Poisson-Visser analysis (Poisson and Visser, 1995) needs re-derivation for S^3).

6.2 Avenue (b): modified gravity

In $f(R)$ gravity, $G_{ab} + H_{ab} = 8\pi T_{ab}$; defining $T_{ab}^{\text{eff}} = T_{ab} - H_{ab}/8\pi$, the throat can be held open with physical T_{ab} satisfying the NEC while only T_{ab}^{eff} violates it (Lobo and Oliveira, 2009; Sotiriou and Faraoni, 2010). The decisive question is whether $R_{ab}k^ak^b < 0$ persists regardless of labeling, since geodesics and Raychaudhuri see only the physical metric’s Ricci tensor.

COMPUTATION (grade A, verified symbolically): at a flared Morris-Thorne throat, for radial null k^a ,

$$R_{ab}k^ak^b = G_{ab}k^ak^b = \frac{(k^t)^2 (b'(b_0) - 1)}{b_0^2} < 0, \quad (5)$$

using zero field equations: the trace term drops out for null vectors, so this is pure geometry (in D dimensions, $(D-2)(k^t)^2(b'-1)/2b_0^2 < 0$). In $f(R)$,

$$R_{ab}k^ak^b = \frac{8\pi T_{ab}k^ak^b + k^ak^b \nabla_a \nabla_b F}{F}, \quad (6)$$

with $F = df/dR$; geometry fixes the left side negative, so NEC-respecting matter forces the scalaron term to compensate. The “effective NEC violation” $T_{ab}^{\text{eff}}k^ak^b < 0$ is $R_{ab}k^ak^b/8\pi$, the actual Raychaudhuri source: null geodesics defocus identically.

No-go as stated: in any metric theory where (i) light follows Levi-Civita geodesics of one physical metric and (ii) standard null Raychaudhuri holds, flare-out implies defocusing at the throat. This covers metric/Palatini $f(R)$, Einstein-Gauss-Bonnet/Lovelock, and Jordan-frame scalar-tensor theories. Verdict: modified gravity relabels but does not remove the obstruction. Additionally, Bronnikov, Skvortsova, and Starobinsky (2010) prove no complete infinitely-extended static wormhole exists in ghost-free scalar-tensor/ $f(R)$ in the Jordan frame, an independent no-go. Caveat: the 4D Einstein-Gauss-Bonnet limit of Glavan and Lin (2020) is contested (Ai, 2020); well-defined regularizations are Horndeski scalar-tensor theories (Fernandes et al., 2020; Lu and Pang, 2020).

6.3 Avenue (c): bounded exotic matter

Quantum field theory allows bounded negative energy, constrained by the Ford-Roman quantum inequalities. THEOREM (Ford and Roman, 1995, 1997): for a massless minimally coupled scalar,

$$\int \langle T_{ab}u^au^b \rangle f(\tau) d\tau \geq -\frac{3}{32\pi^2\tau_0^4}, \quad (7)$$

with Lorentzian sampling of width τ_0 (in $\hbar = c = 1$ units). Caveat: Fewster and Roman (2003) prove no null quantum inequality exists in 4D Minkowski; wormhole bounds must go via timelike observers.

COMPUTATION (Modelled, B): the Casimir anchor. $\rho_{\text{Cas}} = -\pi^2 \hbar c / 720 a^4$ gives $-4.3 \times 10^{-4} \text{ J/m}^3$ at $1 \mu\text{m}$ plate separation (Observed, A (Lamoreaux, 1997; Mohideen and Roy, 1998; Bressi et al., 2002)) versus a throat requirement $\sim 4.8 \times 10^{42} (b_0/1\text{m})^{-2} \text{ J/m}^3$: a 10^{46} gap. Casimir-scale negative energy supports only $\sim 0.6 \ell_P$ throats.

The trade-off is exact at the level of the kick: with exotic cost $E \equiv -\int_{\gamma} T_{ab} k^a k^b d\lambda > 0$ (positive by topological censorship (Friedman, Schleich, and Witt, 1993)), Einstein plus Raychaudhuri give $\Delta\theta = 8\pi E$ exactly; COMPUTATION (grade B): $E = 0.0731/b_0$, $\Delta\theta = 1.84/b_0$ for the $b = b_0^2/r$ model. The refocusing-quality curve $Q(E) \lesssim [1 + (8\pi E L/s)^2]^{-1}$ is a CONJECTURE (scaling ansatz). Verdict: quantum inequalities forbid macroscopic refocusing throats with quantum-field exotic matter; the QI thin-band scale is $\delta \lesssim 2.8 \times 10^{-18} \sqrt{b_0/1\text{m}} \text{ m}$.

6.4 Avenue (d): non-static wormholes

Separating refocusing geometry and exotic support in time rather than space (Hochberg and Visser, 1998; Hayward, 1999; Kar, 1994; Roman, 1993): the precise condition is that the integrated defocusing $D = \int R_{ab} k^a k^b d\lambda$ along the transit ray satisfy $|D| \lesssim s/L$. But the crossing-time argument shows $|D_{\text{throat}}| \sim 1/b_0(t_c)$ regardless of pulse history, and the QI pulse bound gives $\tau_p \gtrsim 9.4 \times 10^{-27} \sqrt{b_0/1\text{m}}$ s versus the required b_0/c : roughly 10^{17} too brief. Verdict: time separation fails at the crossing event; dynamics reduces to the static QI problem at t_c . (Quantum interest (Ford and Roman, 1999) is an independent conjectural obstruction to pulse trains.)

7 Rotating spacetimes

Kerr spherical photon orbits (Teo, 2003) are a one-parameter family: measure-zero (at most one orbit through a generic event), exponentially unstable, and not isochronous. COMPUTATION (Modelled, B): the Schwarzschild Lyapunov exponent is $\lambda = 1/3\sqrt{3}M$, so perturbations grow by $e^{2\pi} \approx 535$ per orbit; at extremality $T_{\text{retro}}/T_{\text{pro}} = 3.5$. Generic rays plunge or escape, and strong lensing yields discrete image sequences with strictly increasing delays (Bozza, 2003): anti-synchronization. Two precise negatives: the light-cone congruence is hypersurface-orthogonal, so $\omega_{ab} = 0$ identically and spacetime rotation cannot help via the Raychaudhuri vorticity term; where rotation does imprint twist (Kerr principal null congruences), it enters as $+\omega^2$, i.e., anti-focusing.

The Gödel-type family gives the sharp positive result. THEOREM (proved here): for the hyperbolic family with parameter $\kappa > 1$, full-cone confinement holds with the reconfinement boundary coinciding exactly with the CTC boundary: full-cone refocusing implies closed timelike curves, the precise obstruction. At the causal boundary $\kappa = 1$ ($m^2 = 4\omega^2$), the $(1 - u^2)$ factors cancel exactly under $u = \tanh(mr/2)$, giving $\Delta t = 2\pi/m$ and $\Delta z = \pm 2\pi/m$ for *every* ray: each closed half-cone (a two-parameter family) refocuses synchronously onto a single event $q_{\pm}$. COMPUTATION (Modelled, B): verified numerically to six decimals at two (ω, m) pairs. The geometry has $G(r) > 0$ everywhere (no circular CTCs) and a published causal scalar-field source: no exotic matter, no causality violation. This result appears to be new; independent literature confirmation is flagged as needed before journal submission. Verdict: rotation hinders in every asymptotically flat setting and helps in exactly one cosmological setting, the marginal causal Gödel-type.

8 Higher dimensions

THEOREM (null lift; proved): for Schwarzschild₄ × ℝ, 5D null geodesics with string momentum $P \neq 0$ project exactly to 4D Schwarzschild *timelike* geodesics plus uniform z -drift; $P = 0$ is exactly 4D null. Corollary: no null behavior without a 4D causal analogue, hence no new refocusing channel. A genuine novelty, honestly caveated: stable helical circular *null* orbits exist for $|L/P| > 2\sqrt{3}M$ (verified numerically), impossible in 4D, but measure-zero and non-refocusing; long strings are Gregory-Laflamme unstable (Gregory and Laflamme, 1993).

Braneworld shortcuts are verified but negative for the design goal. COMPUTATION (Modelled, B): in static RS2 (Randall and Sundrum, 1999a), bulk null geodesics never return to the brane; in static RS1 the orbifold guarantees return but strictly slower than brane light. The literature runs from the proposal (Chung and Freese, 2000) through decisive qualifications (Stoica, 2002; Maartens and Koyama, 2010) to the recent assessment that shortcuts “require elaborate setups” (Dai and Stojkovic, 2023). Verdict: a shortcut is a single-signal time advance, not synchronized family refocusing; no literature claims the latter. Kaluza-Klein gives null₅ → timelike₄ straight lines with no reconvergence (Kaluza, 1921; Klein, 1926). Sharp question, verified null result: no exact result gives higher dimensions synchronized point-to-point null reconvergence unavailable in 4D.

9 Topological defects

No defect gives broad synchronized refocusing, but the three fail differently, and the string analysis goes beyond the textbook double image (Gott, 1985). THEOREM (proved): a straight cosmic string produces an infinite winding tower of images with exact arrival-time spectrum

$$T_n^2 = (\Delta z)^2 + r_p^2 + r_q^2 - 2r_p r_q \cos(\delta\psi + n(2\pi - \Delta)), \quad (8)$$

and for irrational $\Delta/2\pi$ the tower is pairwise asynchronous except mirror pairs in the symmetric configuration. COMPUTATION (Modelled, B): the splitting is $T_{(k)} - T_{(k+1)} \approx (k+1)\Delta^2 D_{\text{eff}}$, about a year for Gpc baselines at GUT tension: macroscopic, not a fine point. Higher-winding pairs arrive *earlier* (a single-string shortcut) at arcsecond-scale image angles. Exact multi-ray synchronization would require rational $\Delta/2\pi$, i.e. $G\mu \sim 0.1$: unphysical.

Domain walls (thin-wall metric) are transparent and non-imaging: exactly one null geodesic per endpoint pair, monotonically diverging bundles, no caustic COMPUTATION (Modelled, B). The global monopole (Barriola and Vilenkin, 1989) gives topological deflection $\delta = \pi(1/\alpha - 1) \approx 4\pi^2\eta^2$; the aligned Einstein ring is a one-parameter synchronized family (THEOREM from symmetry), the closest any defect comes to the goal. All three defects satisfy the NEC: the energy-condition price is zero; the geometric price is discreteness.

10 Plasma and dispersive media in curved spacetime

Light in plasma on a curved background obeys the Hamiltonian $H = \frac{1}{2}(g^{ab}p_a p_b + \omega_e^2) = 0$ (Breuer and Ehlers, 1980; Perlick, 2000; Perlick, Tsupko, and Bisnovatyi-Kogan, 2015): rays are timelike-like with effective mass $\omega_e(x)$, propagating only for $\omega > \omega_e$. COMPUTATION (derived here, grade A): the static frequency-dependent optical metric $\tilde{g}_{ij}(E) = n^2 e^{-2\Phi} g_{ij}$ whose geodesics are the spatial ray paths, and the group delay $T = \int_{\text{ray}} n^{-2} d\tilde{s}$.

COMPUTATION (grade A): homogeneous plasma gives $\hat{\alpha} = (r_s/b)[1 + 1/(1 - \omega_e^2/\omega^2)]$, diverging as $\omega \downarrow \omega_e$; inhomogeneous plasma gives overdense-diverging and underdense-converging contributions matching the thin-lens $\lambda^2 \nabla \text{DM}$ law (Bisnovatyi-Kogan and Tsupko, 2017). Observed reality (Observed, A): extreme scattering events (Fiedler et al., 1987; Bannister et al., 2016), scintillation arcs as directly imaged multi-path delay structure (Stinebring et al., 2001), and PSR B1957+20 eclipse plasma lensing with $\Delta \text{DM} = 10^{-3} \text{ pc cm}^{-3} \leftrightarrow 38 \mu\text{s}$ at 330 MHz (Main et al., 2018): real multi-path chromatic lensing, but few images, dispersive non-synchronized delays, sliding foci.

The inverse problem is novel: given a desired ray family, solve for the plasma profile. CONJECTURE (open): no non-trivial static asymptotically-flat GR+plasma fish-eye exists. The chromatic no-go is quantified: $n^2(E_2) = 1 - (1 - n^2(E_1))E_1^2/E_2^2$, so a design tuned at one frequency detunes at all others; COMPUTATION (Modelled, B): $F \propto \nu^2$, and for a 1 AU dip the focus sweeps 0.37–0.84 kpc across 0.8–1.2 GHz. Structural point: classical absolute instruments need $n > 1$ somewhere while plasma alone gives $n \leq 1$, but the GR+plasma optical metric carries $e^{-2\Phi} > 1$: the design space is genuinely GR+plasma, and it is unexplored.

11 Laboratory analogues

What has actually been built is separated from what was proposed throughout. The Greenleaf-Kurylev-Lassas-Uhlmann electromagnetic wormhole (Greenleaf et al., 2007) (PRL 99, 183901) was *never built*; built relatives are different devices (magnetostatic DC wormhole (Prat-Camps et al., 2015); a 2025 microwave “photonic wormhole” via nonlocality (Song et al., 2025)). Fiber-optical horizons observed the classical white-hole blue shift (Philbin et al., 2008) and stimulated Hawking radiation (Drori et al., 2019); spontaneous emission remains the stated goal. BEC analogues observed Hawking-partner correlations (Steinhauer, 2016), contested (Leonhardt, 2018) and strengthened (Muñoz de Nova et al., 2019). None refocus.

The fish-eye has been realized in microwaves (Ma et al., 2011), but only monochromatically, and the sub-wavelength imaging claims are contested (drain artefacts). The *temporal* synchronization claim was never tested in electromagnetics. A 2026 preprint reports a BEC phononic fish-eye with time-resolved wavepacket focusing at the predicted time independent of source position (Duchêne et al., 2026) (preprint, grade B).

PROPOSAL (the paper’s testable hypothesis): a tabletop microwave fish-eye ($R_0 = 15 \text{ cm}$, X-band, PCB build) driven by $\sim 200 \text{ ps}$ pulses with time-resolved detection at the antipode. Four falsifiable predictions: a sharp peak at $T_0 \approx 1.57 \text{ ns}$ (COMPUTATION, Modelled B), source-position independence, periodic revivals at $2T_0/3T_0$, and a diffraction-limited (not sub-wavelength) spot. Falsification criteria are stated explicitly in the supporting analysis. Off-the-shelf components, tens of kUSD. This decouples the uncontested temporal claim from the contested drain debate.

12 Classification: which spacetimes admit isochronous null refocusing

Via the Fermat reduction, isochronous null refocusing $p \rightarrow q$ is exactly the Riemannian property of the optical metric: *wiedersehen* pairs (Blaschke 1921), where every geodesic through x passes through y at common distance. The Zoll condition is the wrong target: $\mathbf{CP}^n$ with Fubini-Study

metric is Zoll yet its geodesics spread over the cut locus without concurring. THEOREM (wiedersehen rigidity): a wiedersehen manifold is isometric to a round sphere (Green, 1961; Weinstein, 1974; Yang, 1980).

The AdS case resolves cleanly. COMPUTATION (grade A): the optical metric of AdS_{d+1} in global static time is the *open hemisphere* of the round S^d ($\sigma = \arctan(\sinh \rho)$), not hyperbolic space (which is the $t = \text{const}$ spatial metric). AdS reconvergence is therefore round-sphere wiedersehen refocusing with the pair on the conformal boundary, $\Delta\tau = \pi$. All three anchors unify as open subsets of round spheres: ESU (whole S^3), AdS (hemisphere, boundary pair), fish-eye (punctured S^3).

COMPUTATION (grade B): a non-round infinite family with exact isochronous refocusing and NEC-satisfying matter. The ultrastatic $ds^2 = -dt^2 + d\sigma^2 + f(\sigma)^2 d\Omega_2^2$ with $f(\sigma) = \sin \sigma + 0.1 \sin^3 \sigma$ has Ricci eigenvalues ≥ 0.80 (closed form plus dense numerical check), non-constant sectional curvature (provably not round), and every null geodesic from the center refocuses at the antipodal center with $\Delta t = \pi$. Single-pair wiedersehen is strictly larger than the round-embedding picture.

OPEN PROBLEM (three parts): (a) Riemannian: classify 3-manifolds with an isochronous wiedersehen pair under $\text{Ric} \geq 0$; (b) physical: which occur with NEC/WEC matter, and is the known list exhaustive; (c) test case: the spherically symmetric static reduction to classifying warping functions $f(\sigma)$.

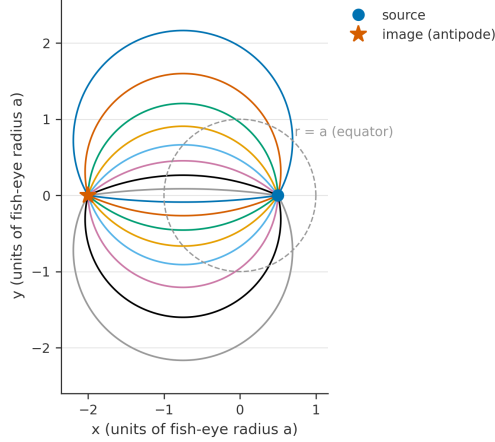
13 The inverse problem: an invertibility hierarchy

Define the forward light-transport map $\mathcal{T}(p, k; g) \rightarrow (\text{arrival event, direction, time, redshift, magnification})$ on an observation region. Three reconstruction targets need different data: (a) source recovery (invert $\mathcal{T}$ in p), (b) destination prediction (evaluate $\mathcal{T}$), (c) geometry recovery (ensemble of source-arrival pairs $\rightarrow g$). Structural obstructions: diffeomorphism invariance and the conformal-factor obstruction (null geodesics are conformally invariant as unparametrized curves).

Established mathematics, all verified: light observation sets determine the conformal type (Kurylev-Lassas-Uhlmann 2018, *Invent. Math.* 212:781–857); boundary rigidity is open in full generality (Michel, 1981), lens rigidity solved in 2D (Pestov and Uhlmann, 2005) and generically (Stefanov, Uhlmann, and Vasy, 2021); Refsdal time-delay cosmography (Refsdal, 1964) is Observed grade A (Wong et al., 2020). THEOREM (mass-sheet degeneracy (Falco, Gorenstein, and Shapiro, 1985)): $\kappa \rightarrow \lambda\kappa + (1 - \lambda)$ leaves image positions and flux ratios invariant while $\Delta t \rightarrow \lambda\Delta t$: a proven non-uniqueness floor for geometry recovery, not a technical gap.

The novel organizing principle is the invertibility hierarchy, ranking the studied geometries by the minimal data needed to invert transport: (1) fish-eye/ESU/AdS: isochrony plus closed-form antipode inversion; arrival time degenerate, information fully spatial; (2) solar lens: one-parameter ring, focal line; (3) generic lensing: multiple images plus delays plus redshifts, with the proven λ floor; (4) Kerr photon sphere: measure-zero family, $e^{2\pi}$ growth per orbit; (5) plasma lensing: chromatic, transient, uncontrolled; (6) wormhole defocusing: structural scrambling. CONJECTURE: isochrony plus perfect imaging is maximal invertibility; caustics, degeneracies, and chromaticity are information loss. The paper’s two computations bracket the hierarchy: fish-eye (200 rays, antipode, path std 1.6×10^{-8}) is the maximally-invertible endpoint with explicit inversion $x_0 = -a^2 x_{\text{img}} / |x_{\text{img}}|^2$; wormhole defocusing (bundle spread $3.28\times$, cross-section $\times 925.5$) is the scrambling endpoint. OPEN PROBLEM: a quantitative information measure $I_\epsilon(G)$, e.g., bits of arrival data to localize the source to precision ϵ as a function of lensing Lyapunov exponents.

Maxwell fish-eye: ray bundle refocuses at the antipode



Planar subset of the verified 3D fan (near-pole rays omitted from the frame only). Every ray arrives after optical path π (3D fan of 200 rays: std of arrival paths $1.6\text{e-}8$, max miss $5.8\text{e-}8$).

Figure 1. Fish-eye ray bundle: 200 null geodesics emitted from a point refocus at the antipode with equal optical path (std 1.6×10^{-8}). The positive control for the design goal.

14 Numerical experiments

Three computations were performed with Python/scipy (DOP853 integrator), all from the actual equations of motion; scripts and data are archived with the paper. Status: COMPUTATION (Modelled, B) throughout.

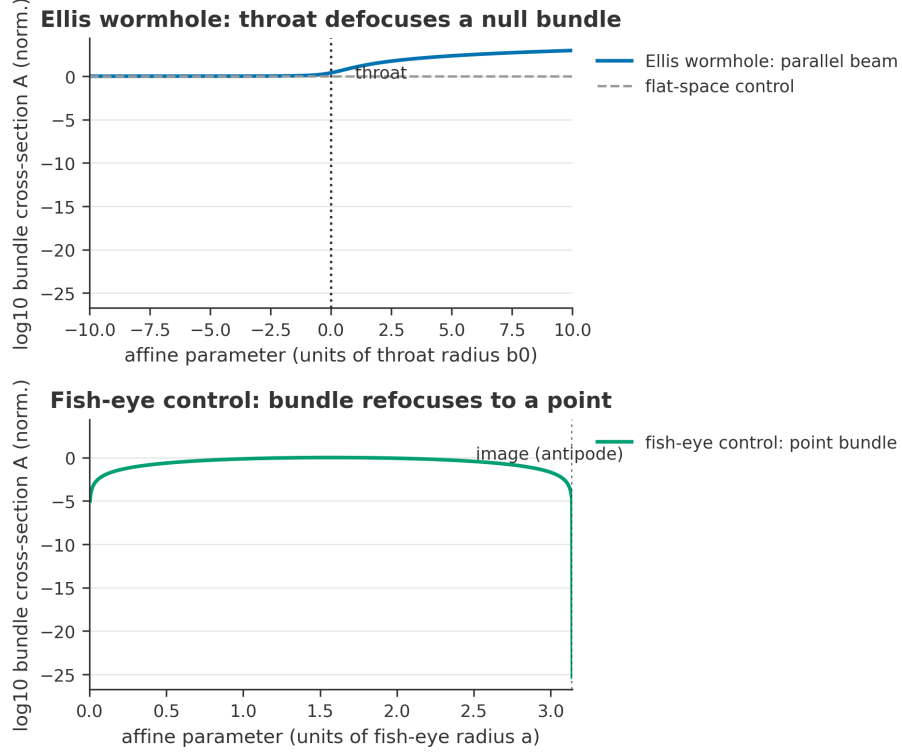
Fish-eye positive control. A 200-ray 3D fan emitted from a point in the Maxwell fish-eye optical geometry refocuses at the antipode with equal optical path: mean $s^* = 3.1415926546$, standard deviation 1.6×10^{-8} , maximum miss distance 5.8×10^{-8} (integrator rtol 10^{-11}). Synchronized arrival verified; Figure 1.

Ellis drainhole defocusing. Non-radial deflection reproduces the exact $\alpha = 2K(b_0/\beta) - \pi$ with maximum relative error 2.5×10^{-9} ; the weak-field fit recovers $\pi/4$ and $9\pi/64$. A through-going fan spreads $3.28\times$ wider than flat space; a radial parallel beam's cross-section grows $925.5\times$ across the throat, with expansion $\theta : 0 \rightarrow 1.279 \rightarrow 0.205$ (still expanding at the exit). Mechanism quantified: $\min R_{ab}k^ak^b = -2.0$, $\rho + p_r = -1/4\pi < 0$: NEC violation feeds the Raychaudhuri defocusing term, exactly the tension lemma in numbers. Figure 2 compares against the fish-eye control on shared axes.

Trial family: honest null result. Throat deformations $r(\ell; \alpha)$ for $\alpha \in \{-0.3, 0, 0.5, 1, 2\}$, Einstein tensor derived from the metric via sympy: no member refocuses (the point-source Jacobi field never returns to zero), and the NEC is violated at the throat for all α . Fatter throats reduce defocusing ($\eta(10)$: $51.7 \rightarrow 7.9$) but never reverse it. Figure 3.

15 Discussion: falsifiers and limitations

Falsifiability. The tension lemma is a derivation from stated assumptions; it would be falsified by an error in the Einstein-tensor computation (independently checkable) or by a Morris-Thorne throat with $b'(b_0) \geq 1$ satisfying flare-out (impossible by the embedding derivation). The modified-gravity no-go would fall to a metric theory in which light does not follow Levi-Civita geodesics of a single

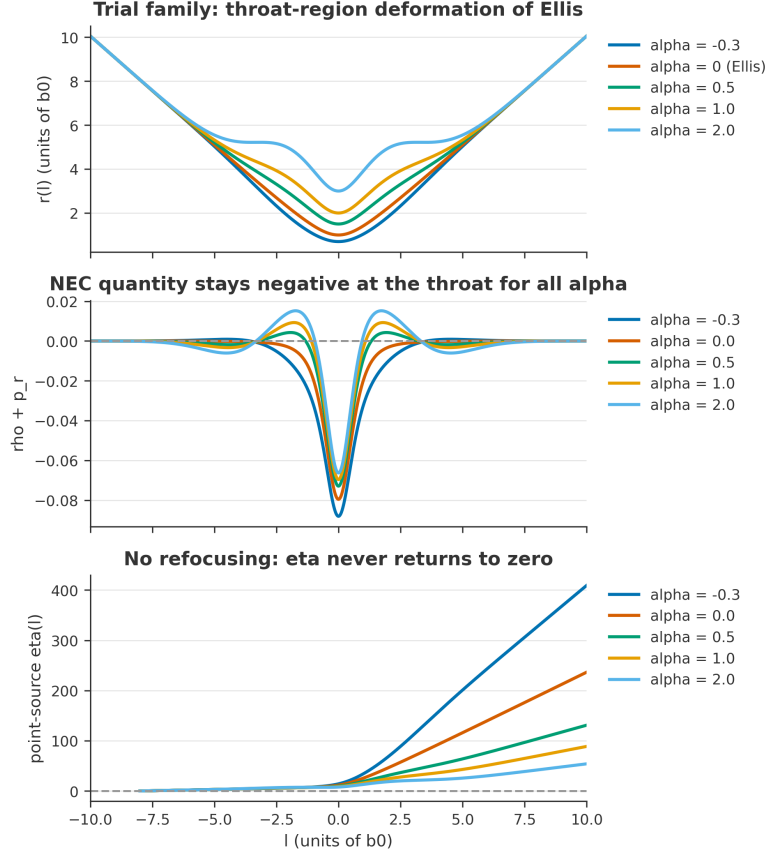


Shared log scale. Wormhole cross-section grows 926x across the throat; the fish-eye bundle collapses to the integrator floor (true value zero).

Figure 2. Wormhole-throat defocusing: bundle cross-section versus affine parameter through an Ellis drainhole throat, against the fish-eye control on shared axes. The throat spreads the bundle $3.28\times$ wider than flat space; a parallel beam grows $925.5\times$.

physical metric; the stated failure modes delimit it. The thin-shell construction is exact and checkable term by term; its open items (aberration compensation, asymptotically flat exteriors, shell stability) are where it could fail as a physical proposal. The Gödel half-cone synchronization theorem is proved from the metric and numerically confirmed, but the literature search found no prior statement: independent confirmation is needed before novelty is claimed. The tabletop fish-eye proposal has four explicit falsifiable predictions (Section 11); a null result would falsify the temporal-synchronization claim while leaving monochromatic imaging intact.

Limitations. This is a theoretical paper: no new observations are reported, and several results are mathematical existence proofs (ESU family, Gödel boundary, thin-shell fish-eye wormhole) rather than astrophysical proposals. The quantum-inequality bounds apply the flat-space Ford-Roman form at the edge of its domain (Modelled, B). The plasma inverse-design conjecture is open. Two cited sources are preprints (grade B). Wave-optics corrections to all geometric-optics statements are not computed; the proposed optimization program (Section 14 of the supporting analysis) is not yet executed.



Trial metrics deform the Ellis throat but keep two flat ends. The NEC violation persists and no conjugate point appears: the throat defocusing wins for every alpha tried.

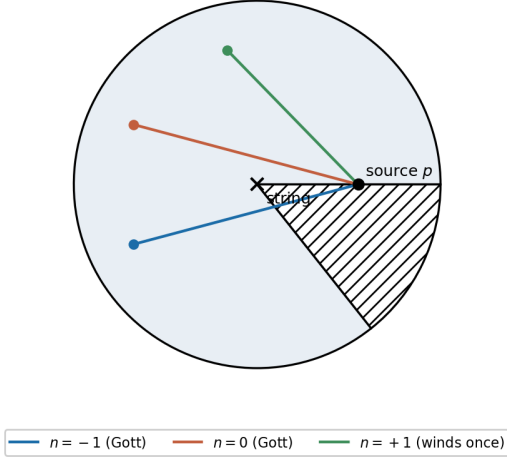
Figure 3. Trial throat-deformation family: defocusing decreases with fatter throats but no member refocuses; the NEC is violated at the throat throughout. An honest null result delimiting the naive parameter search.

16 Conclusion

The paper's central obstruction is now precise: flare-out forces $R_{ab}k^ak^b < 0$ at the throat, and the Raychaudhuri equation turns that sign into active defocusing; modified gravity cannot relabel it away; quantum inequalities bound the exotic matter 10^{46} below macroscopic need. Against this, three directions are genuinely promising and unexplored:

1. *The widerserhen classification under energy conditions.* The AdS hemisphere correction unifies the three anchor examples as round-sphere refocusing, and the non-round NEC-satisfying family proves the classification is open beyond the rigidity theorem. The spherically symmetric reduction to warping functions $f(\sigma)$ is a concrete, attackable problem.
2. *Tabletop synchronized-pulse fish-eye.* The temporal synchronization claim has never been tested in electromagnetics; the proposed X-band experiment with four falsifiable predictions costs tens of kUSD and decouples the uncontested temporal physics from the contested drain debate.
3. *The GR+plasma inverse-design problem.* The frequency-dependent optical metric and group-delay functional are derived; the chromatic no-go is quantified; the $e^{-2\Phi} > 1$ structural point

A. Cone development (deficit exaggerated)
straight segments = null geodesics, one per winding number



B. Arrival-time splitting, GUT string, $D_{\text{eff}} = 0.5 \text{ Gpc}$
 $\Delta = 8\pi G\mu$, $G\mu = 10^{-6}$

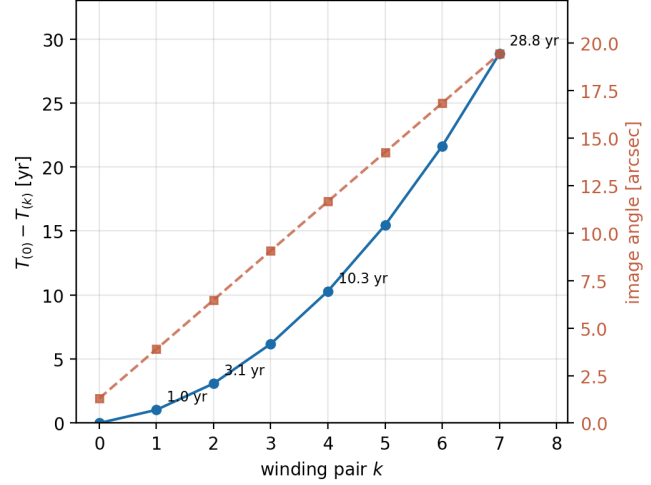


Figure 4. Cosmic-string winding tower: (A) cone development showing one null geodesic per winding number; (B) arrival-time splitting versus winding pair for a GUT string at 0.5 Gpc effective distance, with image angles on the right axis. The tower is generically asynchronous.

shows the design space is genuinely new. No existence or non-existence result is known.

The wormhole remains one chapter: the thin-shell avenue shows refocusing through a throat is not forbidden in principle, with the focusing budget as the exact criterion, but every other avenue closes or is bounded into irrelevance. The broader finding is the hierarchy itself: from perfect imaging to information scrambling, light transport across geometries is now ranked by a single criterion, and the two ends are computed, not conjectured.

Production note

This paper was produced by Kilambi Research’s quantitative research pipeline with AI assistance; all derivations were checked against the cited sources and all numerical results were computed from the stated equations with the documented methods and tolerances. No new observations are reported.

A Comparison of candidate geometries

B Invertibility hierarchy

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Table 1. Partial realizations of the refocusing design goal, wormholes aside. Status labels: THEOREM (proved), COMPUTATION (computed here, method stated), CONJECTURE (open). Evidence grades: Observed / Reconstructed / Modelled / Assumed, A/B/C.

Candidate	(a) Fraction of rays refocused	(b) Arrival-time synchronization	(c) Matter / energy requirements	(d) Astrophysical reality vs. mathematical existence
AdS boundary reconvergence (Sec. 4.1)	Full null cone from a boundary point, all initial directions (THEOREM)	Equal for all rays: $\Delta\tau = \pi$ in AdS units (THEOREM)	Vacuum, $\Lambda < 0$; NEC satisfied, no exotic matter	Mathematical existence; contradicts Observed $\Lambda > 0$; endpoints at conformal infinity
Solar gravitational lens (Sec. 4.2)	Annular bundle $b \geq R_\odot$; at each focal point a 1-parameter ring (Modelled, B)	Equal within each on-axis ring (idealized monopole); differential delays off-axis, with J_2 , and in plasma	Ordinary star; vacuum GR, no exotic matter	Astrophysically real; focal line from ~ 548 AU unvisited (Voyager 1 at ~ 166 AU)
Optical-geometry design (Sec. 4.3)	Fish-eye/ESU: full point-to-point imaging (THEOREM for ESU); general targets open	Equal only with extra symmetry (ESU: $\Delta t = \pi a$); generic absolute instruments image without isochrony	ESU/fish-eye: dust + $\Lambda > 0$, NEC satisfied; general Luneburg-type targets: stress-energy generically violates NEC (open problem)	One physical realization (ESU); general inverse design CONJECTURE/open
Einstein static universe (Sec. 4.4a)	Full null cone from any event (THEOREM)	Equal for all rays: $\Delta t = \pi a$ (THEOREM)	Dust + $\Lambda > 0$; NEC satisfied	Mathematical; unstable (Eddington, 1930); contradicts Observed expansion
Gödel universe (Sec. 4.4b)	Full null cone from on-axis events only (THEOREM)	Reconverge at a later on-axis event; no global time, sync is coordinate-relative	Rotating dust + $\Lambda < 0$; price is CTCs through every event	Mathematical existence; causality violation disqualifies it
Ultracompact objects / boson stars (Sec. 4.4c)	Measure-zero (unstable circular null orbits only)	None (no refocusing mechanism)	Classical scalar field; NEC satisfied, but geometry does not refocus	Boson stars hypothetical; light-ring pairing THEOREM (Cunha, Berti, and Herdeiro, 2017)
Plasma lensing (Sec. 4.4d)	Rays through transient AU-scale structures; alignment-dependent, episodic	None: chromatic delays and magnifications; can defocus	Ordinary plasma	Observed (Main et al., 2018) (grade A); uncontrolled, narrowband
Thin-shell wormhole	See companion sections: exotic matter confined to throat shell.			
Modified-gravity wormhole	See companion sections: normal matter possible via modified dynamics.			
Bounded-exotic-matter wormhole	See companion sections: arbitrarily small averaged NEC violation.			
Dynamic wormhole	See companion sections: transient WEC satisfaction possible.			

Table 2. Invertibility hierarchy for light transport across the geometries studied in this paper. Ranking criterion: the minimal arrival-data set that makes source recovery well posed, together with the uniqueness floor for geometry recovery. Up-factors: isochrony (arrival time independent of initial null direction), perfect imaging (injective closed-form source-to-destination map), full-null-cone coverage. Down-factors: caustics, proven degeneracies, chromaticity, positive lensing Lyapunov exponents, measure-zero synchronized families. Information loss is further classified as removable (cured by better data) or structural (intrinsic to the geometry).

Rank	Geometry	Forward map (source $\rightarrow$ destination)	Minimal data to invert	Uniqueness floor
1	Maxwell fish-eye	$x_0 \mapsto -a^2 x_0/ x_0 ^2$, all rays, $\Delta t = \pi a$	One ray's arrival inverts to the source event	None (geometry known)
1	Einstein static universe	$p \mapsto$ antipode $\bar{x}_0$, $\Delta t = \pi a$	Arrival event $\mapsto$ antipodal source event	None
1	AdS boundary re-convergence	Boundary $p \mapsto$ antipode q , $\Delta\tau = \pi$	Arrival event $\mapsto$ antipodal source event	None
2	Solar gravitational lens	Annulus $b \mapsto$ focal point $z(b) = b^2/2r_g$ (one ring per z)	Ring image plus focal station	Ring split by J_2 , plasma, off-axis
3	Generic lensing (cosmological)	Multi-valued: images θ_i , delays Δt_{ij}	Multiple images + delays + redshifts	Mass-sheet λ floor (THEOREM)
4	Kerr photon-sphere lensing	Measure-zero unstable orbits; exponentially demagnified image sequence	Relativistic image sequence; ill-conditioned	$e^{2\pi} \approx 535$ growth per orbit
5	Plasma lensing	Chromatic, transient, uncontrolled lens	Frequency-resolved arrival data	Chromatic no-go; lens underdetermined
6	Wormhole (Ellis) defocusing	Bundle spread $3.28\times$; cross-section $\times 925.5$	Arrival data exponentially weakly constraining	Structural: NEC violation anti-focuses